Analysis of the Chaotic Regime for DC-DC Converters Under Current-Mode Control

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Abstract - The possibility of chaotic behavior in DC-DC converters under current-mode control has been well established by prior work. Although the spectral modifications that are associated with chaotic operation may provide an important motivation for actual operation in this regime, the literature on chaos in power electronics has tended to treat it more as an exotic effect than as a feasible mode of operation. This may explain why none of the prior work that we are aware of has attempted — for the chaotic regime — to characterize even the most basic property of DC-DC converters, namely the input-output gain (which is the ratio of the average output voltage to the DC input voltage).

The present paper shows how to compute this gain, and other averages of interest, for the chaotic regime of buck, boost, and buck-boost converters under constant-frequency current-mode control and in continuous conduction. Our approach invokes the fact that the chaotic sampled inductor current is *ergodic*, hence governed by a "probability" density, which permits time averages to be replaced by ensemble averages. Although the density can be computed in detail, it turns out that approximating it (quite crudely!) as a uniform density still yields very good results. In contrast, traditional computations based on the nominal (and unstable) periodic solution can be considerably in error.

I. INTRODUCTION

In DC-DC converters under current-mode control, the controller specifies a peak inductor current in each cycle, rather than the duty ratio. For constant-frequency operation, a switch is turned on every T seconds but is turned off when the inductor current reaches a specified reference level, I_{ref}. This reference level is now the primary control variable; the duty ratio D becomes an indirectly controlled auxiliary variable. Steady-state operation with period T and with $D > 0.5$ (approximately) is impossible when I_{ref} is held constant, because this periodic solution is unstable, [1]. Throughout this paper, we examine only the case where I_{ref} is constant, i.e. open-loop operation. The waveforms for $D > 0.5$ under this condition assume complicated forms, corresponding either to periodic operation at some multiple of T (subharmonic operation) or to chaotic variation from cycle to cycle, see for instance [2], [3], [4], [5]. A stabilizing ramp is normally introduced in order to prevent these instabilities and extend the range for stable periodic operation beyond $D > 0.5$.

Although the spectral changes that are associated with chaotic operation may provide an important motivation for actual operation in this regime, the literature on chaos in power electronics has tended to treat it more as an exotic effect than as a feasible mode of operation. This may explain why none of the prior work that we are aware of has attempted, for the chaotic regime, to characterize even the most basic property of such converters, namely the input-output gain (which is the ratio of the average output voltage to the DC input voltage). In view of this, it is no surprise that practitioners implement measures (such as the stabilizing ramp referred to above) to avoid chaotic operation.

In this paper, we show how to compute the input-output gain, and various other averages of interest, for the chaotic regime of DC-DC converters under current-mode control and in continuous-conduction mode. (Computation of spectral characteristics is deferred to a future paper.) Our approach involves recognizing and exploiting the *ergodicity* of the sampled inductor current in a simplified first-order model of the converter. What this means is that the evolution of the inductor current samples is governed by a unique "probability" density. Time averages (whose direct determination would require tedious, costly, and unreliable time-domain simulations) can now be replaced by ensemble averages computed with respect to this density. We demonstrate that very good results are obtained even if the density is approximated as being uniform. In contrast, traditional computations based on the nominal (period-T, unstable) periodic solution can be considerably in error. Our treatment provides convenient analytical expressions to support design for operation in the chaotic regime, and

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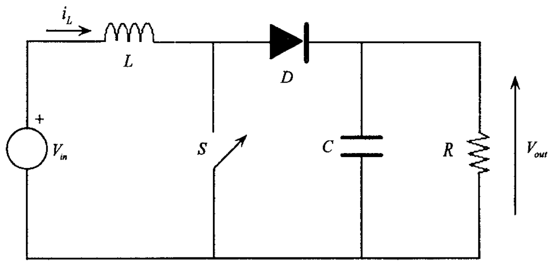


Figure 1: Boost converter circuit.

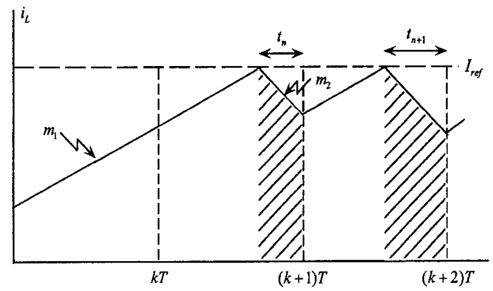


Figure 2: Typical segment of inductor current for chaotic regime, $\alpha > 1$.

thereby enables more serious evaluation of the potential advantages of chaotic operation. Results are presented for buck, boost, and buck-boost converters.

The paper is organized as follows. Section II introduces the role of densities in describing the chaotic regime of a DC-DC converter, using the boost converter as an example. Section III continues to focus on the boost converter, and develops our procedure (previously outlined in [6], but without details) for calculating the output voltage of this converter in terms of the parameters of the circuit. Computer simulations of the converter circuit model, assuming ideal components but not imposing further assumptions (specifically *not* assuming that the inductor current is linear or that the output voltage has negligible ripple) have been used to verify our results, and representative comparisons are presented. Sections IV and V are devoted to similar treatments of the buck-boost and buck converters, respectively.

II. Describing Chaotic Behaviour Via Densities

The boost converter analyzed in this paper is shown in Figure 1. It is assumed that the converter is operating in continuous-conduction mode. The switch is controlled by clock pulses that are spaced T seconds apart. When the switch is closed, the inductor current — driven by V_{in} — increases linearly until it reaches the specified reference value, I_{ref}, at which point the switch opens. Any clock pulse that arrives while the switch is closed is ignored. Once the switch has opened, the next clock pulse causes it to close. Under the assumptions that the output voltage V_{out} is essentially constant and that the switching period is short enough for the inductor current to be essentially piecewise linear, it has been shown in [2] that the dynamics of the converter are well described by the following map:

$$x_{n+1} = 1 - mod_1(\alpha x_n) \tag{1}$$

where: $mod_1(\ )$ denotes multiplication modulo 1 (hence simply the extraction of the fractional part); $x_n = t_n/T$ represents the normalized time from the n-th opening of the switch to the next closing of the switch, as marked in Figure 2; and α is the ratio of the slope magnitudes of the inductor current when the switch is open and closed, respectively:

$$\alpha = \frac{m_2}{m_1} = \frac{(V_{out} - V_{in})/L}{V_{in}/L} = \frac{V_{out} - V_{in}}{V_{in}} \tag{2}$$

The analysis of this equation in [7] shows that for $\alpha < 1$, which corresponds to $D < 0.5$, the equation has a nontrivial stable equilibrium point, corresponding to stable period-T operation of the converter. For $\alpha > 1$, or $D > 0.5$, the equation has no nontrivial stable equilibrium point or stable periodic solution, so stable operation of the converter with period T or any multiple of T is not possible (at least to the extent that the simplified model in (1) actually describes the circuit behavior). For $\alpha = 1$, the solution of the equation for most initial conditions has period 2, which implies the converter has a solution of period $2T$.

Figure 2 represents a typical segment of the inductor current variation with time for $\alpha > 1$. The shaded areas of the figure correspond to intervals when the switch is open; these are precisely the intervals when the diode current is non-zero, equal in value to the inductor current. Unlike in the case of $\alpha < 1$, where the diode current contains one pulse per clock period T, for $\alpha > 1$ there are some periods with no diode current. The ratio of periods with current and no current depends upon the value of α.

The bifurcation diagram for the map (1) is shown in Figure 3, and makes evident the transition from period-T behavior to chaotic operation as α increases through 1. It can be shown, [8], that all points are visited upon iteration for $\alpha \geq \alpha_0$, where $\alpha_0 = (1 + \sqrt{5})/2$ is the golden mean. It has also been shown in [8], citing results from [9], that the map (1) is *ergodic*, which implies the existence — for each α — of a "probability" density $f_\alpha(x)$ that governs the distribution of the values of x_n in the interval [0,1]; the density is termed *invariant*, because it applies for all n. A

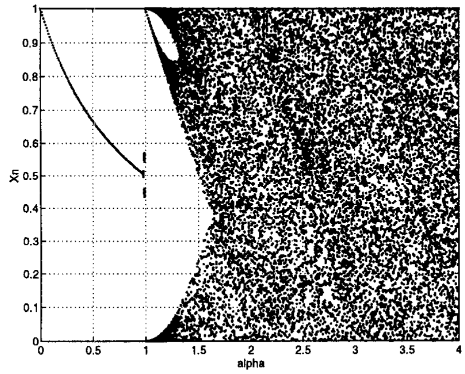


Figure 3: Bifurcation map for (1).

celebrated result of Birkhoff shows that the invariant density can be used to replace time averages of the quantities of interest by ensemble averages, computed with respect to the density, [9]. The density for the map (1) is uniform for integer values of α ($\alpha > 1$), but more generally is piecewise constant, and can be computed in a systematic fashion. In [8] it is shown, for example, that the density for $\alpha = \alpha_0$ (the golden mean) is given by

$$f_{\alpha_0}(x) = \begin{cases} 1/(3-\alpha_0) & for \quad 0 \le x \le 2-\alpha_0 \\ \alpha_0/(3-\alpha_0) & for \quad 2-\alpha_0 < x \le 1 \end{cases} \tag{3}$$

Parallel results have been established in [10], but in the broader and more powerful setting of *eventually-expanding maps*, and *Markov map* approximations for them.

III. Analysis of Chaotic Operation of the Boost Converter

To calculate the output voltage V_{out} (assumed essentially constant) for $\alpha > 1$, we argue as follows. Consider the $(n+1)$-st switching cycle, which is the one that commences with the switch closing at the end of the interval marked t_n in Figure 2, and ends at the next closing of the switch. During this cycle, the capacitor gets some charge through the diode and loses charge through the resistor. The charge through the diode in this cycle is just the shaded area on the right in Figure 2. Under the assumption that V_{out} is essentially constant, this charge is given by

$$Q_D(x_{n+1}) = \left(I_{ref} - \frac{m_2 T}{2} x_{n+1} \right) x_{n+1} T \tag{4}$$

where $m_2 = (V_{out} - V_{in})/L$. The charge lost to the resistor by the capacitor during this switching cycle is $Q_R(x_{n+1}) = p_{n+1} V_{out} T/R$, where p_{n+1} is the number of clock cycles of length T contained in the $(n+1)$-st switching cycle. In the case $\alpha = 2$, it is easy to see that $p_{n+1} = 1$ if $0 \le x_n < 0.5$ and that $p_{n+1} = 2$ if $0.5 \le x_n \le 1$. Hence, for $\alpha = 2$, the net charge into the capacitor in the $(n+1)$-st switching cycle is given by

$$Q_C(x_{n+1}) = \begin{cases} Q_D(x_{n+1}) - V_{out}T/R & 0 \le x_n < 1/2 \\ Q_D(x_{n+1}) - 2V_{out}T/R & 1/2 \le x_n \le 1 \end{cases} \tag{5}$$

More generally, when α is an integer (and $\alpha > 1$), the net charge into the capacitor in the $(n+1)$-st switching cycle is given by

$$Q_C(x_{n+1}) = \begin{cases} Q_D(x_{n+1}) - V_{out}T/R & 0 \le x_n < 1/\alpha \\ Q_D(x_{n+1}) - 2V_{out}T/R & 1/\alpha \le x_n < 2/\alpha \\ \cdots & \cdots \\ Q_D(x_{n+1}) - \alpha V_{out}T/R & \frac{(\alpha-1)}{\alpha} \le x_n \le 1 \end{cases} \tag{6}$$

For the output voltage to remain essentially constant, we require the time-average $\overline{Q_C}$ of $Q_C(x_{n+1})$ to be 0. Invoking the ergodicity of (1), we can replace the time average by an ensemble average $\langle Q_C \rangle$, computed with respect to the appropriate invariant density:

$$\langle Q_C \rangle = \int_0^1 Q_C(x) f_\alpha(x) dx \tag{7}$$

In the case of integer α, the density $f_\alpha(x)$ is uniform, so the required calculation is easy. For this case, evaluating the above integral and equating it to 0 yields the constraint

$$I_{ref} = \frac{(1+\alpha)^2 V_{in}}{R} + \frac{\alpha V_{in} T}{3L} \tag{8}$$

For most non-integer values of α, the determination of $f_\alpha(x)$ for the evaluation of (7) becomes considerably more complicated. However, it turns out that using the constraint in (8) as an approximation for all α ($\alpha > 1$) yields very good results. For instance, when $\alpha = \alpha_0$, we can easily evaluate (7) using (3) and the appropriate generalization of (6) to this non-integer case. The resulting constraint is

$$I_{ref} = \frac{(1+\alpha_0)^2 V_{in}}{R} + \frac{(2\alpha_0 - 1) V_{in} T}{4L} \tag{9}$$

This result is in excellent agreement with the approximate solution (8) for $\alpha = \alpha_0$, because $(\alpha_0/3) \approx (2\alpha_0 - 1)/4$.

To compute V_{out} for a given I_{ref}, we first determine α from (8) and then substitute in (2) to get V_{out}. Figure 4 shows a plot (solid line) of V_{out} in volts as a function of I_{ref} in amps, computed in this way for a particular example. The circuit parameters are $V_{in} = 10$ volts, $R = 20\Omega$, $T = 100\mu$s, $L = 1m$H and $C = 500\mu$F. In Figure 4 we also show a plot (dotted line) of the results computed for this circuit using an accurate second-order sampled-data model, [3],

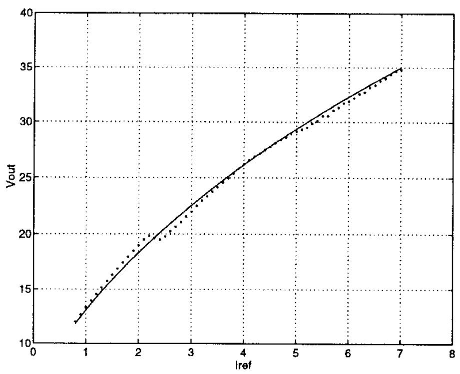


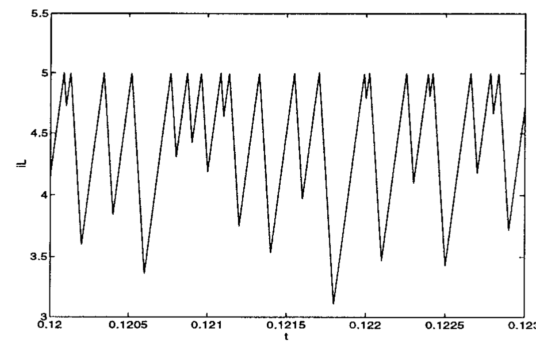
Figure 4: V_{out} in volts as a function of I_{ref} in amps, for the boost converter parameters given in the text. The solid line corresponds to using our approximate expression (8), while the dotted line represents the results of simulations using the sampled-data model of [3].

which tracks samples of the inductor current and capacitor voltage, taken every T seconds. The major discrepancy occurs for $V_{out} = 20$ volts. Since $V_{in} = 10$ volts, this point corresponds to $\alpha = 1$, which is the onset of instability. For values of $\alpha > 1.4$, our approximate results are in excellent agreement with those obtained through the sampled-data simulation.

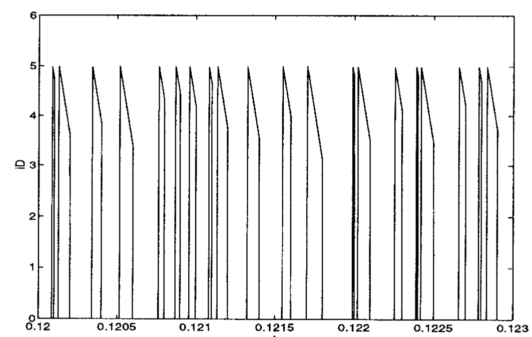
Table 1 shows representative results for this converter as a function of I_{ref} (in amps). The second column is the value of α computed from (8), and the third column gives the value of V_{out} (in volts) obtained by using this value of α in (2). The fourth column, labeled $V_{out,sim}$, shows the average output voltage obtained from simulations of the circuit using SIMULINK from MathWorks (essentially identical results are obtained in SPICE simulations as well). Comparing the third and fourth columns, it is evident that our approximate analysis performs well. Typical waveforms from the SIMULINK simulation are shown in Figure 5. Note that, despite the erratic — chaotic — appearance of the waveforms, the converter is in continuous conduction and the output voltage is essentially constant, with only a small ripple.

The last column in Table 1, labeled $I_{ref,per}$, shows what I_{ref} would be needed in order to obtain the indicated α (and hence V_{out}), *if* the inductor current waveform were *periodic* with period T. Although the periodic solution is *unstable* for the range of α's shown in Table 1, our intent is to see what sorts of results would be obtained if calculations that are routinely done for $D < 0.5$ are blindly extended to $D > 0.5$. Simple calculations show that

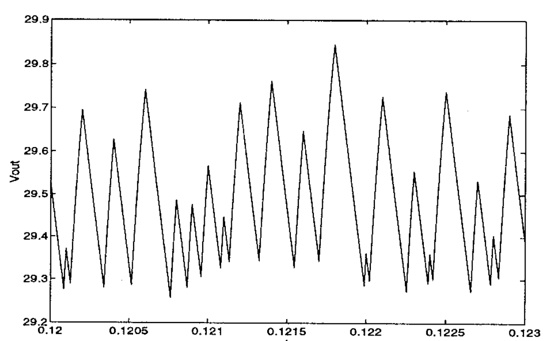
$$I_{ref,per} = \frac{(1+\alpha)^2 V_{in}}{R} + \frac{\alpha V_{in} T}{(1+\alpha)2L} \tag{10}$$



(a)



(b)



(c)

Figure 5: Waveforms of (a) inductor current, (b) diode current, and (c) output voltage, obtained by simulation.

Table 1: Boost converter comparisons.

I_{ref}	α,(8)	V_{out},(2)	$V_{out,sim}$	$I_{ref,per}$
4.0	1.629	26.29	26.2	3.766
5.0	1.95	29.5	29.5	4.682
6.0	2.24	32.41	32.5	5.594
7.0	2.51	35.11	35.2	6.518

It is evident from Table 1 that $I_{ref,per}$ is a poor approximation to the true I_{ref}. This fact indicates the need for a direct analysis (even if approximate) for the chaotic regime, and provides some justification for our efforts.

Another quantity of interest in the analysis of such converters is the duty ratio. The average duty ratio $\langle D \rangle$ in the chaotic regime can be computed in the same way as $\langle Q_C \rangle$ was, using the invariant density. The result has been verified using simulations, and also compared with the duty ratio of $D_{per} = \alpha/(1+\alpha)$ that would be computed for the (unstable) solution of period T. For example, when $\alpha = 2$, computation of the duty ratio for the (unstable) periodic solution yields $D_{per} = 2/3 = 0.667$, while our calculations with the invariant density yield $\langle D \rangle = 0.625$. Similarly, for $\alpha = \alpha_0$, computation of the duty ratio for the (unstable) periodic solution gives $D_{per} = 0.618$, while our calculations yield $\langle D \rangle = 0.602$. (More generally, it turns out that $\langle D \rangle < D_{per}$ throughout the chaotic regime.) In each case, detailed simulations of the circuit, for the converter parameters above as well as for other choices of parameters, have confirmed our results computed from the invariant density.

IV. Buck-Boost Converter

The buck-boost converter is shown in Figure 6 (note the reference polarity we have chosen for V_{out}). The analysis of this circuit is very similar to that of the boost converter. The inductor current waveform again has the appearance in Figure 2, and the dynamics of the converter are still described by the map in equation (1), but with α now given by

$$\alpha = \frac{m_2}{m_1} = \frac{V_{out}/L}{V_{in}/L} = \frac{V_{out}}{V_{in}} \tag{11}$$

To calculate V_{out} for $\alpha > 1$, we follow the same line of reasoning as for the boost converter. The net charge into the capacitor in the $(n+1)$-st switching cycle is given by equation (6), with α given in (11). For the output voltage to be essentially constant, the time average — and hence the ensemble average — of this net charge must be 0. Using the fact that the invariant density is uniform for integer α ($\alpha > 1$), we obtain the following constraint for the integer case:

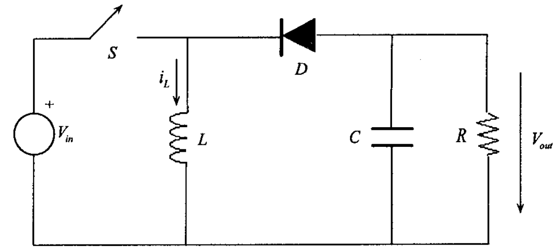


Figure 6: Buck-boost converter circuit.

$$I_{ref} = \frac{\alpha(1+\alpha)V_{in}}{R} + \frac{\alpha V_{in} T}{3L} \tag{12}$$

The analysis is more complicated for other values of α. However, we shall use the result in (12) as an approximation for all $\alpha > 1$. Given I_{ref}, we can compute α from (12) and V_{out} from (11). As a comparison, note that an exact solution of this problem for $\alpha = \alpha_0$, where the density is given by (3), yields the constraint

$$I_{ref} = \frac{(1+2\alpha_0)V_{in}}{R} + \frac{(2\alpha_0 - 1)V_{in}T}{4L} \tag{13}$$

This constraint is in very good agreement with the approximate constraint (12) for $\alpha = \alpha_0$.

Table 2 shows representative results for this converter as a function of I_{ref} (in amps). The parameters of the buck-boost circuit used for this example are $V_{in} = 10$ volts, $R = 20\Omega$, $L = 1$mH, $C = 500\mu$F and $T = 100\mu$s. The second column lists the value of α computed from (12), and the third column gives the value of V_{out} (in volts) computed by using this value of α in (11). The fourth column, labeled $V_{out,sim}$, shows the average output voltage obtained from numerical simulations of the circuit. The last column, labeled $I_{ref,per}$, gives the I_{ref} that would produce the listed value of α (and hence V_{out}) if the inductor current were periodic with period T — an unstable solution. Simple calculations show that

$$I_{ref,per} = \frac{\alpha(1+\alpha)V_{in}}{R} + \frac{\alpha V_{in} T}{(1+\alpha)2L} \tag{14}$$

Once again, we observe significant errors in the predictions obtained by blindly extending the traditional periodic-steady-state analysis into the chaotic regime.

V. Buck Converter

The buck converter circuit is shown in Figure 7. The analysis of this converter follows along the general lines used for the preceding converters, but the fact that the input/output coupling is now made through the inductor

Table 2: Buck-boost converter comparisons.

I_{ref}	α,(12)	V_{out},(11)	$V_{out,sim}$	$I_{ref,per}$
3.667	2	20	19.9	3.333
4.5	2.28	22.8	22.9	4.087
5.5	2.586	25.86	25.7	4.997

rather than the diode requires some changes in the details of the argument. The inductor current waveform again has the appearance in Figure 2, and the dynamics of the converter are still described by the map in equation (1), but with α now given by

$$\alpha = \frac{m_2}{m_1} = \frac{V_{out}/L}{(V_{in} - V_{out})/L} = \frac{V_{out}}{V_{in} - V_{out}} \tag{15}$$

The net charge $Q_C(x_{n+1})$ into the capacitor in the $(n+1)$-st switching cycle is given by an expression of the form (6), except that $Q_D(x_{n+1})$ has to be replaced by $Q_L(x_{n+1})$, the total charge flowing through the inductor in that cycle. Then, when we come to computing the time average and ensemble average — $\overline{Q_C}$ and $\langle Q_C \rangle$ respectively — of $Q_C(x_{n+1})$, what we need are the corresponding time average and ensemble average — $\overline{Q_L}$ and $\langle Q_L \rangle$ respectively — of $Q_L(x_{n+1})$. Now using the fact that the time-averaged power input of the converter must equal the time-averaged power output, and replacing time averages by ensemble averages, we find quite directly that

$$\langle Q_L \rangle = (1 + \alpha)\langle Q_D \rangle \tag{16}$$

where $\langle Q_D \rangle$ is the ensemble-averaged diode current. Note that $Q_D(x_{n+1})$ is still given by the expression in (4), so its ensemble average can be computed exactly as in the earlier cases. Putting all of this together, we can evaluate $\langle Q_C \rangle$ and set it to 0, as before, to obtain the following constraint in the case of integer α ($\alpha > 1$):

$$I_{ref} = \frac{\alpha V_{in}}{(1+\alpha)R} + \frac{\alpha V_{in} T}{(1+\alpha)3L} \tag{17}$$

The constraint turns out to be a very good approximation for non-integer α as well. For instance, an exact solution of this problem for $\alpha = \alpha_0$ using the density in (3) yields

$$I_{ref} = \frac{V_{in}}{\alpha_0 R} + \frac{(3-\alpha_0)V_{in}T}{\alpha_0 4L} \tag{18}$$

which is in excellent agreement with the approximate solution (17) for $\alpha = \alpha_0$.

To find V_{out} for a given I_{ref}, we first compute α from (17) and then solve for V_{out} from (15). Table 3 shows some representative results — under the same categories as in Tables 1 and 2 — for the buck converter. The circuit parameters for our example are $V_{in} = 20$ volts, $R = 10\Omega$,

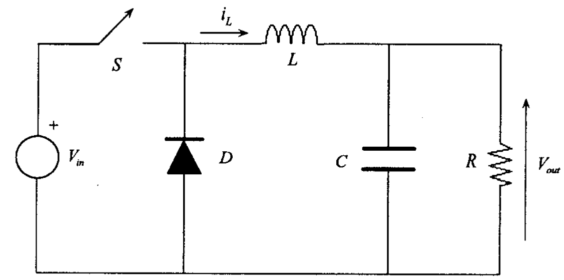


Figure 7: Buck converter circuit.

Table 3: Buck converter comparisons.

I_{ref}	α,(17)	V_{out},(15)	$V_{out,sim}$	$I_{ref,per}$
1.25	1.53	12.1	12.08	1.233
1.3	1.696	12.58	12.6	1.281
1.35	1.884	13.06	13.11	1.329
1.4	2.1	13.55	13.5	1.377

$T = 100\mu$s, $L = 10$mH and $C = 250\mu$F. The expression used to compute $I_{ref,per}$ in the last column, namely the value of I_{ref} that would be needed if the listed α were to be obtained with the (unstable) solution of period T, is

$$I_{ref,per} = \frac{\alpha V_{in}}{(1+\alpha)R} + \frac{\alpha V_{in} T}{(1+\alpha)^2 2L} \tag{19}$$

VI. Summary

This paper has demonstrated how to approach the analysis of DC-DC converters in the chaotic regime, using a state densities approach. We have also presented a simplification that yields excellent approximations and tractable analytical expressions. Finally, it has been established that simple-minded extension of traditional computations from the stable periodic regime into the chaotic regime produces results that can be significantly in error.

Our hope is that the results in this paper can form the basis for more serious exploration of the design implications of operation in the chaotic regime. As the waveforms in Figure 5 make clear, the chaotic regime is not necessarily one to be avoided; although stable period-T operation is lost, the waveforms are still well-behaved, and the output voltage ripple is small. A potential advantage of chaotic operation is that the switching sprectrum is flattened (although at the expense of a corresponding broadening), see for example [5]. As noted in [12], which deals with actively randomized modulation, this spectral shaping may be desirable in some situations. More detailed analytical results on the switching spectrum in chaotic operation will be presented in a future paper.

Several important research issues remain. There are, for example, subtleties involved in making the transition from the simplified equation (1), with its *specified* value of α, to a circuit in which, *under the assumption* that V_{out} is constant, we obtain an *implicitly defined* α. Our simulations, see Figure 5, have shown that the overall picture is consistent, but more remains to be understood. A further research issue is that of closed-loop control in the chaotic regime.

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